

M2-KK6 System in ABJM Theory : Fuzzy S^3 and Wrapped KK6

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ABSTRACT

We study the properties of M2-KK6 (2D membranes - 6D Kaluza-Klein monopole) solution in ABJM membrane theory. First, we find a new kind of BPS solution which has six coordinates, contrasts to our previous solutions which have four coordinates. Next, we argue that, after wrapping 2 sphere the new solution may correspond to the previous solution of four coordinates. We analyze the properties therein and conclude that M2-branes described in ABJM theory could expand into fuzzy three sphere plus a wrapped 2 sphere near the KK6 core. Especially, we show in detail how the fuzzy 3-sphere could arise in these solutions and discuss the property of wrapped KK6 and its relation to M5-brane. We also analyze the fluctuation of the M2-KK6 solution and see that it is U(1) field theory.

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1 Introduction

Bagger, Lambert and Gustavsson (BLG model) [1,2] had proposed a three-dimensional $N = 8$ superconformal Chern-Simons model to describe the low energy effective theory of two coincident M2-branes in eleven dimensions [3]. Use the Nambu bracket algebras [4], which are a infinite-dimensional case of Lie n-algebras, the model could describe infinite M2 branes [5,6].

On the other hand, ABJM [7-10] had proposed an $N = 6$ Chern-Simons-matter theory with $U(N) \times U(N)$ gauge group, $SO(6)$ R-symmetry and equal but opposite Chern-Simons (CS) levels $(k, -k)$, to capture the dynamics of the low-energy limit of multiple M2-branes on M-theory orbifold, C^4/Z_k . An important test for the ABJM multiple membranes theory is that it should reproduce the physics of M2-M5 intersections. The authors in [11,12] had found a kind of solution and argued that it confirms the property. However, a detailed investigation in [13] had found that the solution in [11,12] does not described the M5 but merely a single D4-brane interpretation. Thus the solution in [11,12] is a novel realization of the fuzzy 2-sphere instead of fuzzy 3-sphere.

As is well-known that, in M-theory there is Kaluza-Klein monopole (KK6) object which is a six dimensional object with coordinate $(t, x_1, x_2, z_1, z_2, y_1, y_2)$. It was argued in [14-15] that M2-brane, with coordinate (t, x, z) , can intersect with KK6 over a 0-brane such that one of the M-2-brane coincides with the isometry direction (z) of the Taub-NUT space [14]

$$(0|M2, KK6) = \left\{ \begin{array}{cccccccccccc} t & x & z & - & - & - & - & - & - & - & - & - \\ t & - & (z) & x_1 & x_2 & z_1 & z_2 & y_1 & y_2 & - & - & - \end{array} \right. \quad (1.1)$$

Thus, in considering $(0|M2, KK6)$ we need extra 5 dimensional internal spaces to have 6 space dimensions in the world volume of KK6. This seems to conflict to the conjecture in [11], which said that the funnel of M2-M5 are the only BPS solution in ABJM theory. In this paper we will find a new kind of BPS solution which has 6 coordinates, contrasts to our previously found solutions [16] which have 4 coordinates. This will really enable us to interpret the solution as M2-KK6 system.

For self-consistence we first in section 2 review the ABJM model, then we discuss the BPS solutions with 3 coordinates in [11,12] and solutions with 4 coordinates found in our previous paper [16]. In section 3 we present our new solutions with 6 coordinates. We also calculate the tension of the solution and show how the new solution could describe the M2-KK6 (2D membranes with 6D Kaluza-Klein monopole) system. In section 4 We will argue that, after wrapping 2 sphere the new solution may be

corresponding to the previous solution of 4 coordinates [16]. Using these properties we finally conclude that M2-branes described in ABJM theory could expand into fuzzy three sphere plus a wrapped 2 sphere near the KK6 core. We follow the nice paper of Nastase, Papageorgakis and Ramgoolam [13] to see how the fuzzy three sphere geometry could arise in our new solution. We also analyze the fluctuation on the M2-KK6 solution and see that it is U(1) field theory. We summarize our results in the last section.

2 ABJM Theory and BPS Solutions

2.1 ABJM Theory and BPS Equations

The ABJM theory is an $N = 6$ superconformal $U(N) \times U(N)$ Chern-Simons theory of gauge fields A_μ and \hat{A}_μ with level $(k, -k)$ coupled to four complex scalars Y^A and four Dirac fermions ψ_A , where $A = 1, 2, 3, 4$ in the bifundamental representation [7-10],

$$S = \int d^3x \left[\frac{k}{4\pi} \epsilon^{\mu\nu\lambda} \text{Tr} \left(A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda - \hat{A}_\mu \partial_\nu \hat{A}_\lambda - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda \right) - \text{Tr} D_\mu Y_A^\dagger D^\mu Y^A - i \text{Tr} \psi^{A\dagger} \gamma^\mu D_\mu \psi_A - V_{\text{bos}} - V_{\text{ferm}} \right] \quad (2.1)$$

with the potentials

$$V_{\text{bos}} = -\frac{4\pi^2}{3k^2} \text{Tr} \left(Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger \right), \quad (2.2)$$

$$V_{\text{ferm}} = -\frac{2i\pi}{k} \text{Tr} \left(Y_A^\dagger Y^A \psi^{B\dagger} \psi_B - \psi^{B\dagger} Y^A Y_A^\dagger \psi_B - 2Y_A^\dagger Y^B \psi^{A\dagger} \psi_B + 2\psi^{B\dagger} Y^A Y_B^\dagger \psi_A - \epsilon^{ABCD} Y_A^\dagger \psi_B Y_C^\dagger \psi_D + \epsilon_{ABCD} Y^A \psi^{B\dagger} Y^C \psi^{D\dagger} \right), \quad (2.3)$$

ABJM model actually has $SU(4) \sim SO(6)$ R-symmetry and $\mathcal{N}=6$ supersymmetry.

In considering BPS solution, which have the dependence of only one of the spatial worldvolume coordinate, say “ s ”, the BPS equations of ABJM theory could be obtained by combining the kinetic terms and potential terms in the Hamiltonian and rewriting it as a sum of perfect squares plus some topological terms [12]: If we denote $Y^A = (Z^1, Z^2, W^1, W^2)$ then the formula used in this paper is [12]

$$H = \int dx ds \text{tr} \left(|\partial_s W^{\dagger A} + \frac{2\pi}{k} (W^{\dagger B} W_B W^{\dagger A} - W^{\dagger A} W_B W^{\dagger B} - Z^B Z_B^\dagger W^{\dagger A} + W^{\dagger A} Z_B^\dagger Z^B)|^2 + |\partial_s Z^A + \frac{2\pi}{k} (Z^B Z_B^\dagger Z^A - Z^A Z_B^\dagger Z^B - W^{\dagger B} W_B Z^A + Z^A W_B W^{\dagger B})|^2 \right)$$

$$\begin{aligned}
& + \frac{16\pi^2}{k^2} |\epsilon_{AC} \epsilon^{BD} W_B Z^C W_D|^2 + \frac{16\pi^2}{k^2} |\epsilon^{AC} \epsilon_{BD} Z^B W_C Z^D|^2) \\
& + \frac{\pi}{k} \int dx^1 \text{tr} (W_A W^{\dagger A} W_B W^{\dagger B} - W^{\dagger A} W_A W^{\dagger B} W_B + 2W^{\dagger A} W_A Z^B Z_B^{\dagger} \\
& - 2W_A W^{\dagger A} Z_B^{\dagger} Z^B + Z_A^{\dagger} Z^A Z_B^{\dagger} Z^B - Z^A Z_A^{\dagger} Z^B Z_B^{\dagger}), \tag{2.4}
\end{aligned}$$

in which the last term is topological and doesn't affect the dynamics in the bulk. A set of BPS equations which minimize the energy in a given topological sector is :

$$0 = \partial_s W^{\dagger A} + \frac{2\pi}{k} (W^{\dagger B} W_B W^{\dagger A} - W^{\dagger A} W_B W^{\dagger B} - Z^B Z_B^{\dagger} W^{\dagger A} + W^{\dagger A} Z_B^{\dagger} Z^B) \tag{2.5}$$

$$0 = \partial_s Z^A + \frac{2\pi}{k} (Z^B Z_B^{\dagger} Z^A - Z^A Z_B^{\dagger} Z^B - W^{\dagger B} W_B Z^A + Z^A W_B W^{\dagger B}) \tag{2.6}$$

$$0 = \epsilon_{AC} \epsilon^{BD} W_B Z^C W_D = \epsilon^{AC} \epsilon_{BD} Z^B W_C Z^D \tag{2.7}$$

When the BPS equations are satisfied the topological term in (2.4) gives the energy of the BPS configuration.

2.2 BPS Solution of 3 Coordinates

Assumption that $W^A = 0$ the BPS equation could be reduced to a simple form:

$$\partial_s Z^A + \frac{2\pi}{k} (Z^B Z_B^{\dagger} Z^A - Z^A Z_B^{\dagger} Z^B) = 0, \quad A, B = 1, 2. \tag{2.8}$$

Separate the s -dependent and independent part by

$$Z^A = f(s) G^A, \quad f(s) = \sqrt{\frac{k}{4\pi s}}, \tag{2.9}$$

G^A s are $N \times N$ matrices and shall satisfy the relation

$$G^A = G^B G_B^{\dagger} G^A - G^A G_B^{\dagger} G^B. \tag{2.10}$$

This equation is solved in [11,12] by first diagonalizing G_1^{\dagger} using the $U(N) \times U(N)$ transformations and find that the another matrix G_2^{\dagger} must be off-diagonal. The N dimensional irreducible solution is

$$(G_1^{\dagger})_{m,n} = \sqrt{m-1} \delta_{m,n}, \quad (G_2^{\dagger})_{m,n} = \sqrt{N-m} \delta_{m+1,n}. \tag{2.11}$$

Historically, above solution have two kinds interpretations :

First interpretation : Authors in [11,12] regarded G_1, G_2 as two complex matrix and thus it has four coordinates which can be used to describe M2-M5 system. They had also calculated the tension of the configuration from the above BPS funnel solution

and see that it is consistent with the well known relation between M5-brane and M2-brane tensions : $2\pi T_5 = T_2^2$. To get feeling about the solution we present following forms:

1. The most simple solution of (2.11) is 2×2 matrix

$$X = \text{diag}(0, 1), \quad Y = \text{diag}(0, 0), \quad Z = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad (2.12)$$

It represents funnel solution of 2M2-M5 system [11,12].

2. For the 3×3 matrix the solution of (2.11) is

$$X = \text{diag}(0, 1, \sqrt{2}), \quad Y = 0, \quad Z = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad (2.13)$$

It represents funnel solution of 3M2-M5 system [11,12].

Second interpretation : However, Nastase, Papageorgakis and Ramgoolam [13] had seen that, as $G_1^\dagger = G_1$ the solution in (2.11) will represent the funnel solution with coordinates G_1 , G_2 and G_2^\dagger , which in fact has only 3 coordinates and the funnel solution is fuzzy S^2 instead of S^3 regarded in [11,12]. They had also studied the fluctuations on the solution to convince the property.

2.3 BPS Solution of 4 Coordinates

Denote the matrix notation $Y^A = (X, \tilde{Z}, Y, W)f(s)$ the another BPS solutions with 4 coordinates found in our previous paper [16] are those in the case of $W = 0$. For $N \times N$ matrix with $N = m + n + 1$ the solutions have following non-zero matrix elements:

$$X_{i,i} = (0, 0, \dots, 0, 0, \quad \mathbf{0}, \quad 1, \sqrt{2}, \dots, \sqrt{n-1}, \sqrt{n}) \quad (2.14)$$

$$Y_{i,i} = (\sqrt{m}, \sqrt{m-1}, \dots, \sqrt{2}, 1, \quad \mathbf{0}, \quad 0, 0, \dots, 0, 0) \quad (2.15)$$

$$\tilde{Z}_{i,i-1} = (1, \sqrt{2}, \dots, \sqrt{m-1}, \sqrt{m}, \quad \sqrt{n}, \sqrt{n-1}, \dots, 2, 1) \quad (2.16)$$

Notice that above solution is irreducible as \tilde{Z} matrix does not become a double block forms. To get feeling about the solution we present following forms:

For the 4×4 matrix, with $N=1+2+1$, an irreducible solution is

$$X = \text{diag}(0, \mathbf{0}, 1, \sqrt{2}), \quad Y = \text{diag}(1, \mathbf{0}, 0, 0), \quad \tilde{Z} = \begin{pmatrix} 0 & 1 & & \\ 0 & \mathbf{0} & 0 & 0 \\ & \sqrt{2} & 0 & 0 \\ & 0 & 1 & 0 \end{pmatrix} \quad (2.17)$$

in which $\tilde{Z}_{4 \times 4}$ looks, more or less, like as the overlap of $\tilde{Z}_{2 \times 2}$ matrix in (2.12) with $\tilde{Z}_{3 \times 3}$ matrix in (2.13), with overlapped element $\mathbf{0}$. The property of overlapping therein strongly suggests that this is a irreducible solution.

In [16] we have calculated the tension therein and find an explanation that the solution (2.14)-(2.15) describes $N \times N$ M2 membranes expanding into fuzzy three sphere plus a wrapped 2 sphere near the KK6 core. This is along the first interpretation in section 2.2, in which we regard each of X , Y and \tilde{Z} as independent complex matrix. However, if we adopt the second interpretation in section 2.2, then we have only four coordinates X , Y , \tilde{Z} and \tilde{Z}^\dagger .

It is the work of this paper to find BPS solution which does has 6 coordinates which really enable us to describe M2-KK6 system. Especially, we show in detail how the fuzzy 3-sphere could arise in these solutions and discuss the property of wrapped KK6 and its relation to M5-brane.

3 New BPS Solutions

3.1 BPS Solution of 6 Coordinates : Pseudo-Direct-Product Solution

Denote the notation $Y^A = (X, Z, Y, W)f(s)$ for $N \times N$ matrix with $N = m + n + 1$, the new BPS solutions with 6 coordinates have following non-zero matrix elements:

$$X_{i,i} = (0, 0, \dots, 0, 0, 0, \mathbf{0}, 1, \sqrt{2}, \sqrt{3}, \dots, \sqrt{n-1}, \sqrt{n}) \quad (3.1)$$

$$Y_{i,i} = i(\sqrt{m}, \sqrt{m-1}, \dots, \sqrt{3}, \sqrt{2}, 1, \mathbf{0}, 0, 0, 0, \dots, 0, 0) \quad (3.2)$$

$$Z_{i,i-1} = (0, 0, 0, \dots, 0, 0, 0, \sqrt{n}, \sqrt{n-1}, \dots, \sqrt{3}, \sqrt{2}, 1) \quad (3.3)$$

$$W_{i,i+1} = i(1, \sqrt{2}, \sqrt{3}, \dots, \sqrt{m-1}, \sqrt{m}, 0, 0, 0, 0, \dots, 0, 0) \quad (3.4)$$

Notice that above solution is irreducible as Z and W matrix do not become a double block forms. To get feeling about the solution we present following forms:

For the 4×4 matrix, with $4=1+2+1$, an irreducible solution is

$$X = \text{diag}(0, \mathbf{0}, 1, \sqrt{2}), \quad Z = \begin{pmatrix} 0 & 0 & & \\ 0 & \mathbf{0} & 0 & 0 \\ & \sqrt{2} & 0 & 0 \\ & 0 & 1 & 0 \end{pmatrix} \quad (3.5)$$

$$Y = i \text{ diag}(1, \mathbf{0}, 0, 0), \quad W = i \begin{pmatrix} 0 & 1 & & & \\ 0 & \mathbf{0} & 0 & 0 & \\ & 0 & 0 & 0 & \\ & 0 & 0 & 0 & \end{pmatrix} \quad (3.6)$$

Let us make important comments on the above solution.

1. Matrix $Z_{4 \times 4}$ looks, like as the overlap of $Z_{3 \times 3}$ matrix in (2.13) with $0_{2 \times 2}$, with overlapped element $\mathbf{0}$. Also matrix $W_{4 \times 4}$ looks like as the overlap of $Z_{2 \times 2}$ matrix in (2.12) with $0_{3 \times 3}$, with overlapped element $\mathbf{0}$.

2. It is easily to see that the matrix with above X and Z while taking $Y = W = 0$ is an BPS solution. As it is corresponding to 3-coordinates BPS solution in section 2.2. Similarly, matrix with above Y and W while taking $X = Z = 0$ is also an BPS solution.

3. If we have a solution with $(X, Z, 0, 0)_{m \times m}$ and another solution with $(0, 0, Y, W)_{n \times n}$ then we can combine them to become a direct product matrix $(X, Z, Y, W)_{(m+n) \times (m+n)}$. This new matrix is surely a solution. But it is reducible from a direct-product matrix solution.

4. Although our new solution looks very similar to a direct product solution it is irreducible however. The crucial point is that there is an **overlapped element “0”** as can be explicitly seen in (3.5) and (3.6). We call this solution as **pseudo-direct-product solution**. Above solution is a pseudo-direct-product solution of two matrix $M_{2 \times 2}$ and $N_{3 \times 3}$.

For more clear we present following irreducible 6×6 matrix solution, with $N=2+3+1$.

$$X = \text{diag}(0, 0, \mathbf{0}, 1, \sqrt{2}, \sqrt{3}), \quad Z = \begin{pmatrix} 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & & \\ 0 & 0 & \mathbf{0} & 0 & 0 & \\ & & \sqrt{3} & 0 & 0 & 0 \\ & & 0 & \sqrt{2} & 0 & 0 \\ & & 0 & 0 & 1 & 0 \end{pmatrix} \quad (3.7)$$

$$Y = i \text{ diag}(\sqrt{2}, 1, \mathbf{0}, 0, 0, 0), \quad W = i \begin{pmatrix} 0 & 1 & 0 & & & \\ 0 & 0 & \sqrt{2} & & & \\ 0 & 0 & \mathbf{0} & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \end{pmatrix} \quad (3.8)$$

This a **pseudo-direct-product solution** of two matrix $M_{3 \times 3}$ and $N_{3 \times 4}$.

3.2 Proof

We now prove that the matrix X , Z , Y and Z in (3.1)-(3.4) is the solution of BPS equations (2.5)-(2.7).

• Step 1:

Use (3.1)-(3.2) we see that $XY = YX = 0$. This implies

$$\Rightarrow \epsilon^{BD} W_B X W_D = Y X W - W X Y = 0 \quad (3.9)$$

$$\Rightarrow \epsilon^{BD} Z_B Y Z_D = X Y Z - Z Y X = 0 \quad (3.10)$$

• Step 2 :

Use (3.3) and (3.4) we see that $ZW = WZ = 0$. This implies

$$\Rightarrow \epsilon^{BD} W_B Z W_D = Y Z W - W Z Y = 0 \quad (3.11)$$

$$\Rightarrow \epsilon^{BD} W_B W W_D = X W Z - Z W X = 0 \quad (3.12)$$

Thus the BPS equations (2.7) are satisfied.

• Step 3:

Use (3.1)-(3.4) we see that $W^{\dagger B} W_B X = W^{\dagger B} W_B Z = X W_B W^{\dagger B} = Z W_B W^{\dagger B} = 0$.

Thus the BPS equation (2.6) is reduced to be

$$\begin{aligned} 0 &= \partial_s Z^A + \frac{2\pi}{k} (Z^B Z_B^\dagger Z^A - Z^A Z_B^\dagger Z^B - W^{\dagger B} W_B Z^A + Z^A W_B W^{\dagger B}) \\ &= \partial_s Z^A + \frac{2\pi}{k} (Z^B Z_B^\dagger Z^A - Z^A Z_B^\dagger Z^B) \end{aligned} \quad (3.13)$$

This equation is just BPS equation in (2.8) and solution of X and Z in (3.1) and (3.3) is just the BPS solution of 3 coordinates in (2.11) plus a direct product of zero matrix.

• Step 4 :

Use (3.1)-(3.4) we see that $Z^B Z_B^\dagger Y^\dagger = Z^B Z_B^\dagger W^\dagger = Y^\dagger Z_B^\dagger Z^B = W^\dagger Z_B^\dagger Z^B = 0$. Thus the BPS equation (2.5) is reduced to be

$$\begin{aligned} 0 &= \partial_s W^{\dagger A} + \frac{2\pi}{k} (W^{\dagger B} W_B W^{\dagger A} - W^{\dagger A} W_B W^{\dagger B} - Z^B Z_B^\dagger W^{\dagger A} + W^{\dagger A} Z_B^\dagger Z^B) \\ &= \partial_s W^{\dagger A} + \frac{2\pi}{k} (W^{\dagger B} W_B W^{\dagger A} - W^{\dagger A} W_B W^{\dagger B}) \end{aligned} \quad (3.14)$$

This equation is just BPS equation in (2.8) and solution of Y and W in (3.2) and (3.4) is just the BPS solution of 3 coordinates in (2.11) plus a direct product of zero matrix.

Q.E.D.

3.3 Energy of BPS Solutions and KK6 Tension in ABJM

The energy of the solution can be evaluated from (2.4).

$$\begin{aligned}
E &= \frac{\pi}{k} \int dx \operatorname{tr}(W_A W^{\dagger A} W_B W^{\dagger B} - W^{\dagger A} W_A W^{\dagger B} W_B + 2W^{\dagger A} W_A Z^B Z_B^{\dagger} \\
&\quad - 2W_A W^{\dagger A} Z_B^{\dagger} Z^B + Z_A^{\dagger} Z^A Z_B^{\dagger} Z^B - Z^A Z_A^{\dagger} Z^B Z_B^{\dagger}) \\
&= 2 \int ds dx \operatorname{tr}(\partial_s Z_A^{\dagger} \partial_s Z^A + \partial_s W_A^{\dagger} \partial_s W^A) = \frac{k}{8\pi} (m(m+1) + n(n+1)) \int ds dx s^{-3} \\
&= \frac{k}{8\pi} \begin{cases} \frac{N^2-1}{2} \int ds dx s^{-3}, & \text{odd } N \\ \frac{N^2}{2} \int ds dx s^{-3}, & \text{even } N \end{cases} \quad (3.15)
\end{aligned}$$

Note that we have used the BPS equations (2.5) and (2.6) to obtain the second line. In the last line we consider only the lowest energy case among these solution for a fixed value of $N = m + n + 1$. Now, introducing the dimension parameter of M2 tension T_{M2} [12] we can define the radius R averaged over the M2 by

$$R^2 = \frac{2\operatorname{tr}(Y^A Y_A^{\dagger})}{N} = \begin{cases} \frac{N^2-1}{N} \frac{k}{4\pi T_{M2}} \frac{1}{s}, & \text{odd } N \\ N \frac{k}{4\pi T_{M2}} \frac{1}{s}, & \text{even } N \end{cases} \quad (3.16)$$

In terms of R the energy becomes

$$E = \begin{cases} \frac{T_{M2}^2}{\pi} \frac{N^2-1}{N^2} \int dx dR \frac{R^3}{k} 2\pi^2 &= \frac{T_{M2}^2}{\pi} \frac{N^2-1}{N^2} \int dx d^4 y \frac{1}{k}, & \text{odd } N \\ \frac{T_{M2}^2}{\pi} \int dx dR \frac{R^3}{k} 2\pi^2 &= \frac{T_{M2}^2}{\pi} \int dx d^4 y \frac{1}{k}, & \text{even } N \end{cases} \quad (3.17)$$

To proceed we shall notice that in our solution the values of $X, Y, Z, W, Z^{\dagger}, W^{\dagger}$ are non-zero and we have six coordinates. However, the radius defined in (3.14) will always produce the $dR R^3$ in the energy. This will always produce the volume integration $dx d^4 y$ which, at first sight, could not provide a sufficient space dimension, six, to represent the M2-KK6 system.

To solve the puzzle let us first notice that M2-brane can intersect with KK6 over a 0-brane such that one of the worldvolume with coordinate (t, x, z) of the M-2-brane coincides with the isometry direction “ z ” of the Taub-NUT space [14] (see (1.1)). In this case M2 is a wrapped configuration [15]. Thus the integrations $\int dx$ in (3.17) shall be taken over a wrapped value of $\int dx = g_s \ell_s$. (Note that $R_{11} = g_s \ell_s$) and (3.17) becomes

$$\begin{aligned}
E &= \frac{T_{M2}^2}{\pi} \frac{N^2-1}{N^2} g_s \ell_s \int \frac{d^4 y}{k} = \frac{T_{M2}^2}{\pi} \frac{N^2-1}{N^2} g_s \ell_s \cdot \frac{1}{4\pi \ell_s^2} \cdot \int \frac{d^6 y}{k} \\
&= \frac{N^2-1}{N^2} T_{KK6} \int \frac{d^6 y}{k}, \quad \text{odd } N \quad (3.18)
\end{aligned}$$

$$\begin{aligned}
E &= \frac{T_{M2}^2}{\pi} g_s \ell_s \int \frac{d^4 y}{k} = \frac{T_{M2}^2}{\pi} g_s \ell_s \cdot \frac{1}{4\pi \ell_s^2} \cdot \int \frac{d^6 y}{k} \\
&= T_{KK6} \int \frac{d^6 y}{k}, \quad \text{even } N
\end{aligned} \tag{3.19}$$

in which we have added two sphere volume $\int dy^2 = \int_{\ell_s} d\Omega_2 = 4\pi \ell_s^2$. This result is just that in [16].

This means that the KK6 in M2-KK6 system described in ABJM theory shall be wrapped with 2 sphere with radius ℓ_s . Thus M2 membranes could expand into fuzzy S^3 plus a wrapped 2 sphere (with radius ℓ_2) near the KK6 core. In this interpretation we have a KK6 from M2 in ABJM theory. We now detail the property in next section.

4 Fuzzy S^3 , Wrapped KK6 and 2M5

We discuss some properties of above new BPS solution.

4.1 Fuzzy S^3 in Six Coordinates Solution

In this subsection we investigate the geometry of new solution following the paper of Nastase, Papageorgakis and Ramgoolam [13].

First, we can easily calculate the following $Y_A Y_B^\dagger$ bilinears, in which $Y_A = (X, Z)$:

$$(XX^\dagger)_{i,i} = (0, \dots, 0, \mathbf{0}, 1, 2, 3, \dots, n-1, n) \tag{4.1}$$

$$(ZZ^\dagger)_{i,i} = (0, \dots, 0, \mathbf{0}, m, m-1, m-2, \dots, 1) \tag{4.2}$$

$$(XZ^\dagger)_{i,i+1} = (0, \dots, 0, 0, \sqrt{m-1}, \sqrt{2}\sqrt{m-2}, \sqrt{3}\sqrt{m-3}, \dots, \sqrt{m-1}) \tag{4.3}$$

$$(ZX^\dagger)_{i,i-1} = (0, \dots, 0, 0, \sqrt{m-1}, \sqrt{2}\sqrt{m-2}, \sqrt{3}\sqrt{m-3}, \dots, \sqrt{m-1}) \tag{4.4}$$

If we denote $XX^\dagger \equiv J_1^1$, $XZ^\dagger \equiv J_1^2$, $ZX^\dagger \equiv J_2^1$, $ZZ^\dagger \equiv J_2^2$ and define $J_i = (\sigma_i)_\alpha^\beta J_\beta^\alpha$ then we find the following algebra

$$[J_i, J_j] = 2i\epsilon_{ijk} J_k \tag{4.5}$$

which shows a $SU(2)$ symmetry group, as was checked in [13] for 3 coordinate solution.

Next, we can easily calculate the following $Y_A Y_B^\dagger$ bilinears, in which $Y_A = (Y, W)$:

$$(YY^\dagger)_{i,i} = (n, n-1, \dots, 2, 1, \mathbf{0}, 0, 0, \dots, 0, 0) \tag{4.6}$$

$$(WW^\dagger)_{i,i} = (1, 2, \dots, n-1, n, \mathbf{0}, 0, 0, \dots, 0, 0) \tag{4.7}$$

$$(YW^\dagger)_{i,i-1} = (\sqrt{n-1}, \sqrt{n-2}, \dots, 1, 0, 0, 0, \dots, 0, 0) \tag{4.8}$$

$$(WY^\dagger)_{i,i+1} = (\sqrt{n-1}, \sqrt{n-2}, \dots, 1, 0, 0, 0, \dots, 0, 0) \tag{4.9}$$

If we denote $YY^\dagger \equiv \tilde{J}_1^1$, $YW^\dagger \equiv \tilde{J}_1^2$, $WY^\dagger \equiv \tilde{J}_2^1$, $WWZ^\dagger \equiv \tilde{J}_2^2$ and define $\tilde{J}_i = (\sigma_i)_\alpha^\beta J_\beta^\alpha$ then we find the following algebra

$$[\tilde{J}_i, \tilde{J}_j] = 2i\epsilon_{ijk}\tilde{J}_k \quad (4.10)$$

which constitute another $SU'(2)$ symmetry group as that in (4.5).

Finally, use (4.1)-(4.4) and (4.6)-(4.9) we can check the following relation

$$[J_i, \tilde{J}_j] = 0 \quad (4.11)$$

Thus we obtain the $SU(2) \times SU'(2)$ form $Y_A Y_B^\dagger$ bilinears. As fuzzy S^3 has symmetry $SO(4) \sim SU(2) \times SU'(2)$ our new solution thus shows a fuzzy S^3 structure. Let us make following comments to conclude this section.

1. Although the appearance of symmetry group $SU(2) \times SU'(2)$ is trivial in direct product solution, it is **not so trivial** in the pseudo-direct-product solution. The commutation relation (4.11) is crucial to make sure the property.

2. We can use bilinears $(X^\dagger X)_{i,i}$, $(Z^\dagger Z)_{i,i}$, $(X^\dagger Z)_{i,i-1}$ and $(Z^\dagger X)_{i,i+1}$ to form another $SU(2)$ symmetry group, in addition to (4.1)-(4.5). In a similar way, We can use bilinears $(Y^\dagger Y)_{i,i}$, $(W^\dagger W)_{i,i}$, $(Y^\dagger W)_{i,i-1}$ and $(W^\dagger Y)_{i,i+1}$ to form another $SU(2)$ symmetry group, in addition to (4.6)-(4.10). However, these symmetry merely reflects the bifundamental represent in ABJM theory and is irrelevant to the fuzzy geometry in funnel solution, as detail in [13].

4.2 Wrapped KK6

The BPS solution of 4 coordinates in (2.14)-(2.16) found by us in a previous paper [16] also implies tension relation (3.15). How can we explain this “coincidence” ?

First, we see that add matrix Z in (3.5) to iW^\dagger in (3.6) will becomes matrix \tilde{Z} in (2.17), i.e.

$$Z[eq.(3.5)] + iW^\dagger[eq.(3.6)] = \tilde{Z}[eq.(2.17)] \quad (4.12)$$

In fact, this is a general property between the 6-coordinates solution (3.1)-(3.4) and 4-coordinates solution (2.14)-(2.16). This property is related to the wrapping procedure in KK6. To see the physical reason behind it, let us consider a system with the 2 coordinate (x,y). We can also use another 2D coordinates (“x+y”, “x-y”) to describe this 2D coordinate (x,y). Now, if we wrap a coordinate “x-y” then it remains only

coordinate “ $x+y$ ”. Thus, for a 6 coordinates $(x_1, x_2, x_3, x_4, x_5, x_6)$ system the 4 coordinates $(x_1, x_2, “x_3 + x_4”, “x_5 + x_6”)$ system represents the original one while with double wrapped coordinate.

In this interpretation, we conclude that our previous solution (2.14)-(2.16) [16] is just a double wrapped of new solution (3.1)-(3.4).

We shall remark that although one can drop any coordinate at hand, it may be nonsense. In fact, only if the remained coordinates are also the system solution can we drop some coordinates to describe a physical system.

4.3 Fuzzy S^3 in four Coordinates Solution

Note that the BPS solution of 3 coordinates has only $SU(2)$ symmetry which corresponds to fuzzy S^2 , as detailed in [13]. However, to correspond to Fuzzy S^3 geometry, which is $SO(4)$ symmetry, we need merely 4 coordinate [17] which is less than the 6 coordinates in our new solution. So, what is happen here?

The reason behind this is that we need to wrap 2 coordinate as mentioned in our interpretation in section 3.3. Thus, after wrap 2 coordinate we remain 4 coordinate which may corresponds to fuzzy S^3 . This then lead to two interesting problems.

1. Does the symmetry group in 6-coordinates solution is larger than fuzzy S^3 ? It seems that it is, but we have not yet found the symmetry group.
2. Does it appears fuzzy S^3 in 4-coordinates solution ? The answer is yes. This is because that use (4.12) we can “split” the “one” \tilde{Z} coordinate into “two” coordinates of Z and W . In this way, as that from section 4.1 we can obtain the fuzzy S^3 therefore.

Finally, we shall remark that although one can “split” any coordinate into summation of two arbitrary coordinates, it may be nonsense. In fact, in our “splitting” the final two coordinates are also the BPS solution of ABJM theory. Thus, only if the split coordinates are also the system solution can we study the property of original solution from the split system. In this interpretation, we can find the hidden symmetry in original system (4-coordinates solution) from the split system (6-coordinates solution with fuzzy S^3).

4.4 Fluctuation

We now consider the fluctuations around the new solution following the paper of Nastase, Papageorgakis and Ramgoolam [13]. The investigations of fluctuation on fuzzy S^3 are more involved. The key point is that we first take a small fluctuation around the

BPS solution of coordinate Y_A . For example, the fluctuation of matrix $Y(s)_\alpha$ around the solution Y_α^0 may be expressed as a general form

$$Y(s)_\alpha = f(s)Y_\alpha^0 + r_\alpha \quad (4.13)$$

in which r_α is the fluctuation field and contains U(1) field [13]. We then expand the ABJM Lagrangian and identify

$$[J_i,] \rightarrow \epsilon_{ijk} x_j \partial_k \quad (4.14)$$

in which J_i are defined in section 4.1. Now, after the calculations it is found that the fluctuation on fuzzy $S^2 \sim SU(2)$ will show U(1) field [13]. As the case in [13] has only 3 coordinates while our solutions have 6 coordinates it is straightforward to see that the fluctuation on fuzzy $S^3 \sim SU(2) \times SU'(2)$ will also show U(1) field while on the KK6 worldvolume. Thus the field theory KK6 worldvolume is U(1) theory. This is consistent with the analysis in [15].

4.5 Wrapped KK6 and 2M5

Use the relation $2T_5 = \frac{T_2^2}{\pi}$ the (3.17) tells us that the 4 coordinates solution can be interpretation as M2-2M5 solution.

If this is right, then the worldvolume theory of KK6, which is U(1) field theory, shall be the worldvolume theory of 2 M5, which is expected to be non-Abelian self-dual 2 form field theory [18]. Then, they contradict to each other.

To explain the contradiction let us see the configuration in (4.15) and remind of a fact that, when we wrap coordinates y_1 and y_1 in KK6 the result configuration does not correspond to M5.

$$\begin{array}{llllllllllll} M2 & : & t & x & z & - & - & - & - & - & - & - \\ KK6 & : & t & - & (z) & x_1 & x_2 & z_1 & z_2 & y_1 & y_2 & - \\ M5 & : & t & - & z & x_1 & x_2 & z_1 & z_2 & - & - & - \end{array} \quad (4.15)$$

It is known that, after wrap coordinates z the non-Abelian self-dual 2 form field theory will be reduced to gauge field theory. Thus, up to the quadratic fluctuation it just give U(1) type Lagrangian, that is described in section 4.4

Note that the correspondence between 4-coordinates solution of M2-2M5 and 6-coordinates solution of M2-KK6 seems to enable us to find non-Abelian self-dual 2 form field theory by studying fluctuation on M2-KK6 system. In fact, the isometry direction in KK6 lead us to wrap a direction on M5, and this will render self-dual 2 form field theory to be one-form gauge field theory.

In conclusion, we have following summarizations:

1. 3-coordinates solution found in [11,12] shows fuzzy S^2 as studied in [13].
2. Wrapped isometry in 4-coordinates solution (M2-2M5) = Wrapped 2 sphere in 6-coordinates solution (M2-KK6). It shows fuzzy S^3 .
3. In order to obtain (M2-2M5) system from (M2-KK6) system we shall first wrap 2 sphere in (M2-KK6) system and then “open” the isometry coordinate “z”. It shall be emphasized that, without “open” the isometry coordinate we could not obtain (M2-2M5) system.

Finally, the reader may wonder how can “two” M5 to form “one” KK6. Let us explain the “combination” as following.

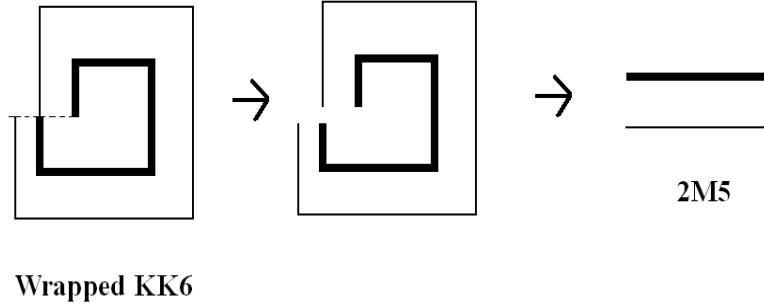


Figure 1. “One” wrapped KK6 produces “two” M5.

Let us see the figure 1. We first consider “one” KK6 wrapped two times around a circle, $0 < \theta \leq 2\pi$ (left diagram). Next, we “open” the isometry coordinate and cut it at $\theta = 0$ (middle diagram). Finally, we extend the two objects and then obtain “two” M5 (right diagram).

According this explanation 2M5 are just double free M5-branes while without interaction between them. Thus, the corresponding worldvolume field theory on KK6 is free U(1) theory.

5 Discussion

In this paper we find a new kind of BPS solution which has six coordinates, contrasts to our previous solutions which have four coordinates [16]. We calculate the tension of the solution and show a possible way to explain our solution as describing the M2-

KK6 (2D membranes with 6D Kaluza-Klein monopole) systems. We argue that, after wrapping 2 sphere the new solution may correspond to the previous solution of four coordinates. Using these properties we conclude that M2-branes described in ABJM theory could expand into fuzzy three sphere plus a wrapped 2 sphere near the KK6 core. We following the Nastase, Papageorgakis and Ramgoolam [13] to see how the fuzzy three sphere geometry could arise in our new solutions. Especially, we discuss the property of wrapped KK6 and its relation to M5-brane. We also analyze the fluctuation on the M2-KK6 solution and see that it is U(1) field theory.

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